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Note

Generating plans in linear logic* II. A geometry of conjunctive actions

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Abstract

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After the proof-theoretic study given in Masseron et al. (1991), we propose a new geometric characterization of actions, in the spirit of Girard's proof nets and Bibel's connection graphs, in the conjunctive case.

1. Introduction

Time is not an extra datum but is a *sine qua non* condition of change! Indeed, "permanence in time" is a perfectly meaningful expression but "change in time" often used in the literature is symptomatic of a relative misunderstanding of both the concepts. So, we think a pertinent translation of "change" has to imply an interpretation of time succession, that is, an ordering. In [8] we have mentioned that a proof in a linear theory can be provided with an ordering which is induced by the nonsymmetry of the cut rule; our aim is to make precise and exploit this remark.

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General convention. Linear orderings are not realistic at all and we will always consider *partial orderings*, without further mention.

1.1. General considerations about plans

A proof, after Gentzen [4], contains nonconstructive information, which we are now going to eliminate in order to obtain diagrams similar to Bibel's ones [1–3]: our diagrams will be given an orientation, which is a very precious piece of information that will turn out to be the key of our theorem.

The method is analogous to the one used by Girard in [5] for his proof nets: firstly, we define a class of graphs equipped with an orientation $\leqslant$, called *pseudo-plans.* Secondly, we associate a pseudo-plan with every formal action, which is said to be its *plan*: the theorem gives an intrinsic property for a pseudo-plan to be a plan. Another source of inspiration is [6], as is evident from the title.

Let us give the general lines of the construction of plans.

We construct the plans, by induction over the proofs, with the following devices:

- A logical axiom gives an empty plan.
- A transition axiom gives a plan reduced to a single vertex, as shown below.
- When a $\otimes$_r rule is applied, the new plan is obtained by set-theoretic union of the initial ones.
- When a cut rule is applied, the new plan is obtained by plugging the second initial plan into the first one: the links introduced by this operation permit one to define an orientation denoted by $\leqslant$.
- Another rule does not give rise to any transformation.

We now can express our result as follows.

Theorem on plans. *A pseudo-plan whose set of entries is a part of the initial state is a plan iff its orientation $\leqslant$ is an ordering.*

2. Plans: conjunctive case

Definitions. A *pseudo-plan* is a finite graph composed of vertices and oriented edges:

- Each *vertex* is labelled by the name of a transition axiom (of course, the same axiom can generate several labels); a vertex labelled by the axiom $A_1, \ldots, A_m \vdash B_1 \otimes \cdots \otimes B_n$ is provided with the entries $\mathrm{i}A_1, \ldots, \mathrm{i}A_m$ and the exits $\mathrm{x}B_1, \ldots, \mathrm{x}B_n$ (see Fig. 1).
- An *oriented edge* admits an exit $\mathrm{x}A$ as its origin and an entry of the same type $\mathrm{i}A$ as its end.

The *entries and exits of a pseudo-plan* are the ones which are not the origin or the end of an edge.

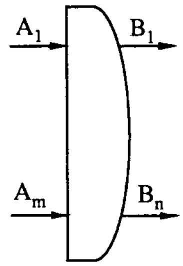

Fig. 1.

The *orientation of a pseudo-plan* is the transitive closure of the relation defined over the set of all entries and exits by
- $\mathrm{i}A < \mathrm{x}B$ if $\mathrm{i}A$ and $\mathrm{x}B$ are attached to the same vertex, and
- $\mathrm{x}A < \mathrm{i}A$ if there exists an edge between $\mathrm{x}A$ and $\mathrm{i}A$.

This defines as well a relation between the vertices, on the basis of: $\mathscr{E} \leqslant \mathscr{F}$ if and only if there exist $\mathrm{x}A \in \mathscr{E}$ and $\mathrm{i}B \in \mathscr{F}$ such that $\mathrm{x}A < \mathrm{i}B$.

One constructs a pseudo-plan $\mathscr{D}$ associated with a proof $\boldsymbol{D}$ by induction over the proofs:
- If $\boldsymbol{D}$ is a proper axiom, $\mathscr{D}$ is reduced to a vertex, labelled by an occurrence of the name of that axiom, provided with the entries and exits corresponding to it.
- If $\boldsymbol{D}$ is another kind of axiom, $\mathscr{D}$ is empty.
- If $\boldsymbol{D}$ is obtained from $\boldsymbol{E}$ by an application of the rule $\otimes\_l$ rule, $\mathscr{D}$ is identical to $\mathscr{E}$.
- If $\boldsymbol{D}$ is obtained from $\boldsymbol{E}$ and $\boldsymbol{F}$ by an application of the $\otimes\_r$ rule, $\mathscr{D}$ is the union of $\mathscr{E}$ and $\mathscr{F}$.
- If $\boldsymbol{D}$ is obtained from $\boldsymbol{E}$ and $\boldsymbol{F}$ by an application of the cut rule on the formula $B_1 \otimes \cdots \otimes B_n$, $\mathscr{D}$ is obtained from the union of $\mathscr{E}$ and $\mathscr{F}$ by linking $\mathrm{x}B_j$ to $\mathrm{i}B_j$ for each j such that $\mathrm{x}B_j$ is an exit of $\mathscr{E}$ and $\mathrm{i}B_j$ is an entry of $\mathscr{F}$.

Remarks.
- In the latter case, there may be no link added to the union: from the action point of view, this corresponds to an independence condition; from the proof-theoretic point of view, it can be interpreted as the elimination of a logical cut, which is an interesting property of the present construction.
- It is clear that two distinct formal actions may have the same plan.
- One checks that the entries of the pseudo-plan constructed from a proof of the sequent $\Gamma \vdash A$ correspond to some atomic components of Γ and that its exits correspond to some atomic components of A.

Definition. A *plan* is a pseudo-plan constructed from a formal action.

Fig. 2 gives the plan associated with the formal action of our "Blocks world" example of [8, Section 6].

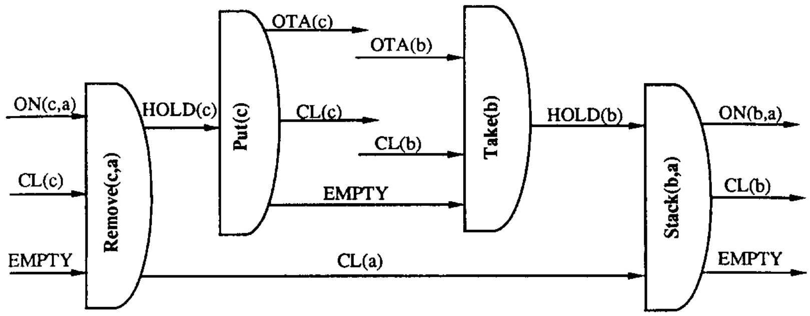

Fig. 2.

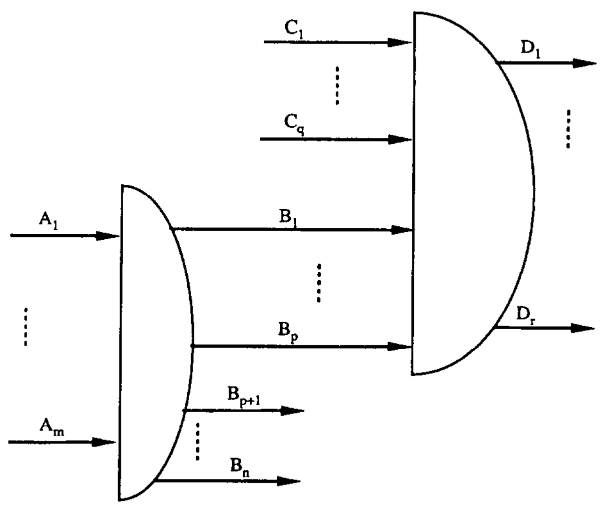

Fig. 3.

2.1. *Demonstration of the theorem*

The construction of a plan is obviously given an orientation which is an ordering.

Conversely, let $\mathscr{D}$ be a pseudo-plan whose orientation is an ordering. Let us choose a vertex which is minimal with respect to $\leqslant$ over $\mathscr{D}$: this minimality condition entails the absence of any edge entering this vertex; thus, if we cut off the edges which go out of this vertex, we obtain the pseudo-plan $\mathscr{E}$ reduced to a single vertex that corresponds to the proper axiom

$$A_1, \ldots, A_m \vdash B_1 \otimes \cdots \otimes B_n$$

and the pseudo-plan $\mathscr{F}$ containing the other vertices and edges. Let us draw up the statement of accounts for the entries and exits:

– All entries $\mathrm{i}A_1, \ldots, \mathrm{i}A_m$ of $\mathscr{E}$ are entries of $\mathscr{D}$ (this is still a matter of minimality).

- By giving, if necessary, new indices to the B_j, one can designate by $xB_1, \ldots, xB_p$ the exits of $\mathscr{E}$ freed by the cutting off of the edges, and by $xB_{p+1}, \ldots, xB_n$ the ones that already were exits of $\mathscr{D}$.
- In the same way, let us designate by $iC_1, \ldots, iC_q$ the other entries and by $D = D_1 \otimes \cdots \otimes D_r$ the conjunction of the formulae of the remaining exits of $\mathscr{D}$ (see Fig. 3).

The induction hypothesis applies to both pseudo-plans: the first one gives the formal action reduced to the concerned axiom, the second one gives a formal action leading to the sequent $B_1, \ldots, B_p, C_1, \ldots, C_q \vdash D$: one then concludes with the method used in the demonstration of the completeness property for the formal actions of [8] (essentially a cut): it enables us to construct a proof of the sequent which we were aiming at.

3. Conclusion

We have translated the conjunctive actions into plans, which are perfectly characterized. The general case of disjunctive actions, theoretically more complex, will be treated in a forthcoming paper.

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